Delay-Lock Tracking of Stochastic Signals

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Abstract—The measurement and tracking of the delay between two versions of a stochastic signal by cross-correlation techniques is considered. Such techniques have broad applications, e.g., interferometry, noncontact speed and distance measurement, etc.

The paper begins by discussing the functional diagram of the tracking system. From this diagram a mathematically equivalent model of the system is derived and its similarities to the well known baseband model of the phase-locked loop are discussed. Using Fokker-Planck (F-P) techniques the performance of the system, as a function of fundamental system parameters, is computed and graphically illustrated. These results are then compared with experimental results obtained by computer simulation.

Nonsense

I. INTRODUCTION

ESTIMATING and tracking the delay between two versions of the same signal is encountered in many fields such as radar, sonar, radio astronomy, ranging, etc.

Another less known application is precise noncontact speed and distance measurement by cross-correlation techniques [1], [2]. The basic principle of a system using correlation techniques to measure velocity is shown in Fig. 1. The light from two sources, located a distance L apart on a vehicle is focused on the surface, reflected and converted by two photodetectors into two electric signals. Due to random surface irregularities the photo-detected signals are random, ideally both will be identical apart from a delay T which is related to the velocity by $T = L/v$. The problem of measuring v is therefore equivalent to measuring T. The unknown delay T could be obtained in principle by delaying the leading signal $x(t)$ by $\hat{T}$ and by correlating the two signals $x(t - T)$ and $x(t - \hat{T})$. The correlation function $R_{x,x}(T - \hat{T}) = E[x(t - T)x(t - \hat{T})]$ has its maximum for $T - \hat{T} = 0$ and a maximum seeking correlator could be used to track $T(t)$.

Since the location of the maximum of $R_{x,x}(T - \hat{T})$ is of interest only, the two signals $x(t - T)$ and $x(t - \hat{T})$ are passed through a linear, physically realizable filter pair $H_1(\omega)$, $H_2(\omega)$ (see Fig. 2). The purpose of this filter pair is to generate an odd correlation function

$$E[y_1(t)y_2(t)] = R_{y_1,y_2}(\phi = T - \hat{T}) = -R_{y_1,y_2}(-\phi). \quad (1)$$

The zero value of $R_{y_1,y_2}(\phi)$ can be tracked using simple feedback techniques as illustrated by Fig. 2.

The same tracking device (tracker) is found in many other areas [3]-[5] and is therefore of rather general interest.

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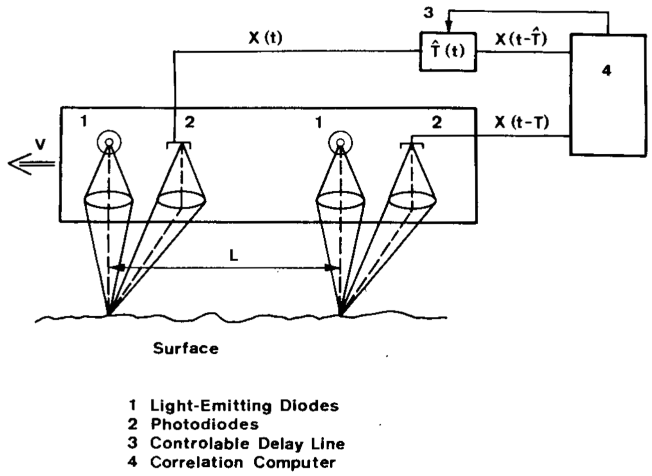


Fig. 1. Noncontact speed measurement by correlation techniques.

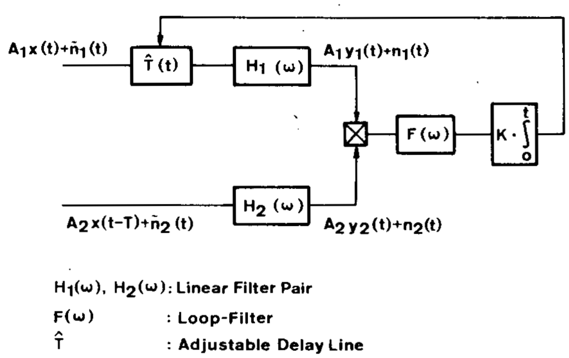


Fig. 2. Block diagram of a DLL.

In this paper we first derive a mathematically equivalent model for an analog-digital (A/D) hybrid version of the tracker shown in Fig. 3. A tracker used as noncontact speedometer was realized in this manner. The main idea leading to this model is based on the decomposition of the multiplier output $z(t;\phi)$ in two parts,

$$z(t;\phi) \equiv E[z(t;\phi)] + \{z(t;\phi) - E[z(t;\phi)]\}. \tag{2}$$

The expected value $E[z(t;\phi)]$ is the useful signal to control the delay line $\hat{T}$ while the mean free part

$$n(t;\phi) = z(t;\phi) - E[z(t;\phi)] \tag{3}$$

can be considered as noise. Note that $n(t;\phi)$ never vanishes—even if the two signals $n_1(t)$ and $n_2(t)$ are identical to zero—since it contains a term which is solely dependent on the product $y_1(t) \cdot y_2(t)$. In the sequel this component is referred to as intrinsic noise.

The behavior of the A/D hybrid tracker is described by a nonlinear difference equation which is approximated by a differential equation in order to be able to use the powerful Fokker-Planck (F-P) method. The equivalent model[1] corresponding to this differential equation has much similarity to the well known baseband model of the phase-locked loop (PLL). There are, however, two important differences; 1) the nonlinearity is aperiodic and 2) the noise process depends on the state $\phi = T - \hat{T}$. The dependence of the noise process $n(t;\phi)$ on ϕ which is due to the intrinsic noise can be neglected for most communications applications, however, for certain applications the intrinsic noise is of central importance and limits the usefulness of application of such systems [2].

[1] As will be demonstrated, this model is identical to the one obtained for a delay-locked loop (DLL) with locally generated reference, although the instrumentation of the latter is completely different.

Subsequently we derive an F-P equation for the system to be analyzed. The system obeys a differential equation of the form

$$\frac{dx}{dt} = f(x) + \sigma(x)\xi(t) \tag{4}$$

where $\xi(t)$ is a stationary process with finite correlation time, hence, for every sample function (4) is an ordinary differential equation. Following an approach outlined by Stratonowitch in [6, p. 89 ff] an approximate F-P equation for (4) assumes the form

$$\frac{\partial}{\partial t} p(x;t) + \frac{\partial}{\partial x}\left\{\left[f(x) + \frac{1}{4}\frac{d}{dx}\sigma^2(x)\right] p(x;t) - \frac{1}{2}\frac{\partial^2}{\partial x^2}\left[\sigma^2(x)p(x;t)\right]\right\} = 0.$$

Note that the term $\frac{1}{4}(d/dx)\sigma^2(x)$—which is equivalent to the Itô-correction term—is lost if $\xi(t)$ is formally replaced by a white noise process. Having derived an F-P equation we analyze in detail a first-order tracker. For many applications this is by far the most important case in practice which is quite the contrast to trackers such as PLL's. The approach outlined in this paper can, however, also be extended to higher order loops. We then show that the variance $\sigma^2{}_\phi$ of the phase error diverges in the stationary state for any physically realizable system. This is due to the fact that the loop falls out of lock (similar to cycle slipping in PLL's) which we refer to as failure. Therefore the tracker has to be supervised by a lock detector. As demonstrated in this paper the actual behavior of the combined tracker-lock detector system can be modeled by a renewal Markov process. The stationary pdf and the mean time between two successive failures are obtained by solving one F-P equation of the renewal process. Finally, the theoretical results are shown to agree very well with computer simulation results thus confirming the validity of the approximations which lead to the model.

II. SYSTEM EQUATIONS

A. Configuration of the Tracker

The block diagram of an A/D hybrid tracker is shown in Fig. 3. As the figure shows, the tracker is a nonlinear feedback system employing a linear filter pair $H_1(z)$, $H_2(z)$ to generate the odd correlation function, A/D and digital-analog (D/A) converters and an adjustable delay line. The delay of samples is easily accomplished with the aid of a shift register driven with variable clock frequency f_p. In the following we approximate

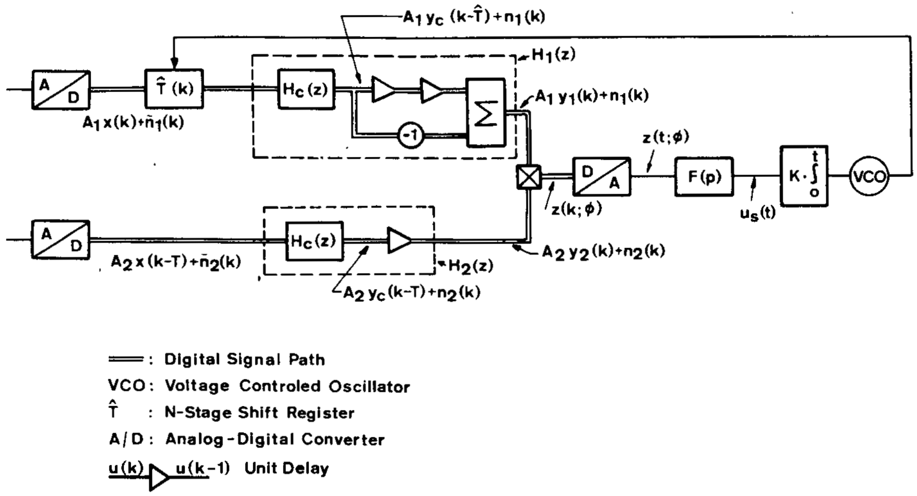


Fig. 3. A/D DLL with symmetric decomposition of $H_1(z)\cdot H_2(1/z)$.

the shift register by its continuous analog with the control law

$$\frac{d\hat{T}}{dt} = Ku_s(t) \tag{5}$$

K: constant.

This is permissible if we assume $1/fp \ll \hat{T}$ and $d\hat{T}/dt \ll 1$. The dynamical behavior of the tracker can now be described by means of differential equations instead of difference equations thus permitting the use of the powerful F-P method.

In practice one usually finds an automatic gain control (AGC) in both signal paths; for subsequent analysis we will assume that the signal power is constant.

$$P_1 = A_1{}^2 \qquad P_2 = A_2{}^2; \qquad E[x^2(t)] = 1. \tag{6}$$

II. GENERATION OF AN ODD CORRELATION FUNCTION

Let $x(k)$ be a wide sense stationary random sequence and assume $\hat{T},T$ to be constant for the moment. If and only if the z transforms of the digital filters $H_1(z)$, $H_2(z)$ fulfil the relation

$$H_1(z)H_2(1/z) = -H_1(1/z)H_2(z) \tag{7}$$

then it is shown in [7] that the cross-correlation function is odd, i.e.,

$$R_{y_1,y_2}(-\phi) = -R_{y_1,y_2}(\phi) \tag{8}$$
$$\phi = T - \hat{T}.$$

The decomposition of the poles and zeros of the rational function $H_1(z)\cdot H_2(1/z)$ is not unique. One especially advantageous decomposition leads to the relation exhibited in Fig. 3. The signals in both channels are passed through the same filter $H_c(z)$. The poles and zeros of this filter can be used to synthesize an optimum nonlinearity. The odd correlation function is then generated by a filter with $H(z) = z^2 - 1$ in the upper channel and with $H(z) = z$ in the lower channel. For $H_c(z) = 1$ we find in the time domain

$$\begin{aligned} R_{y_1,y_2}(\phi) &= E[A_1y_1(k)A_2y_2(k)] \\ &= E\{[-A_1y_c(k-\hat{T}) \\ &\quad + A_1y_c(k-2-\hat{T})]\,A_2y_c(k-1-T)\} \\ &= A_1A_2[-R_{yc,yc}(\phi+1) + R_{yc,yc}(\phi-1)]. \end{aligned} \tag{9}$$[2]

Thus, in the simplest case, we only have to compute two points of the autocorrelation function $R_{x,x}(\phi)$ which are $\frac{1}{2}f_p$ apart. Due to its simplicity this solution is often found in practice [2], [5].

III. MEAN VALUE, POWER SPECTRUM, AND INTENSITY COEFFICIENT OF THE PRODUCT PROCESS $z(t;\phi)$

In the following we will assume that the two additive noise signals $n_1(t)$, $n_2(t)$ (see Fig. 2) are statistically independent of each other and the signal $x(t)$ and have zero mean value. The (time dependent) expected value of $z(t;\phi)$ is determined by the statistics of $x(t)$—which is assumed to be wide sense stationary—the filter pair $H_1(z)$, $H_2(z)$ and the dynamics of $T(t)$ and $\hat{T}(t)$, respectively. In all practical cases $\phi(t)$ is a slowly varying, quasistationary process the bandwidth being much smaller than that of the process $z(t;\phi)$. Under these circumstances $z(t;\phi)$ can be therefore treated as if it were wide sense stationary. The expected value of the time-discrete process $z(k;\phi)$ is given by (see Fig. 3)

$$E[z(k;\phi)] = A_1A_2[-R_{y_c,y_c}(\phi+1) + R_{y_c,y_c}(\phi-1)]. \tag{10}$$

It will prove useful to write (10) as a product of a normalized function $g(\phi)$ and a constant A.

$$Ag(\phi) = A_1A_2[-R_{yc,yc}(\phi+1) + R_{yc,yc}(\phi-1)] \tag{11}$$

with

[2] $R_{y_c,y_c}(\phi+1)$ means the value of the correlation function taken at $\tau = \phi + 1/f_p$ where ϕ is a continuous variable and $1/f_p$ is the time between two clocks.

$|g(\phi)| \leqslant 1.$

Later on in our study we will have to know the power spectrum $S_{n,n}(\omega;\ \phi)$ where $n(k;\ \phi)$ is defined as the mean free noise process

$$n(k;\phi) = z(k;\phi) - E[z(k;\phi)]\,. \tag{12}$$

We show in Appendix A that the power spectrum $S_{n,n}(\omega;\ \phi)$ can be written as the sum of two functions

$$S_{n,n}(\omega;\phi) = S_{n_n,n_n}(\omega) + S_{n_i,n_i}(\omega;\phi). \tag{13}$$

This is a consequence of the fact that $n(k;\ \phi)$ consists of two components of entirely different nature. The first term of (13) describes the influence of the additive noise processes. It vanishes for $n_1(k),\ n_2(k) = 0$. The function $S_{n_i,n_i}(\omega;\ \phi)$ is the power spectrum of the so-called intrinsic or self noise, caused by the signals $y_1(k), y_2(k)$ itself. It is important to realize that the power spectrum $S_{n_n,n_n}(\omega)$ of the additive noise processes is independent of ϕ while the one of the intrinsic noise is strongly dependent on ϕ.

Of particular interest is the value of the power spectrum in the vicinity of zero since this part of the spectrum cannot be eliminated. As shown in Appendix A $S_{n_i,n_i}(\omega;\ \phi)$ vanishes at $\omega = 0$ if $\phi = 0$. This is true, if and only if, the product $\hat{H}_1(z)\cdot H_2(1/z)$ is decomposed as shown in Fig. 3. Since $\phi = 0$ is the stable lock point of the loop (assuming $dT/dt \leqslant 1$) it is a very desirable property that the power spectrum of the intrinsic noise vanishes at $\omega = 0$.

Example 1: Let the correlation function $R_{y_c,y_c}(\phi)$ have the triangular form of Fig. 4. The resulting odd correlation function for the DLL shown in Fig. 3 is illustrated by Fig. 4(b). The power spectrum $S_{n_i,n_i}(\omega;\ \phi)$ for several values of x is shown in Fig. 5. The intensity coefficient K_{n_i} defined by

$$K_{n_i}(x) = S_{n_i,n_i}(\omega = 0; x) \tag{14}$$

plays an important role in the development of the theory of tracking systems and is plotted in Fig. 6.

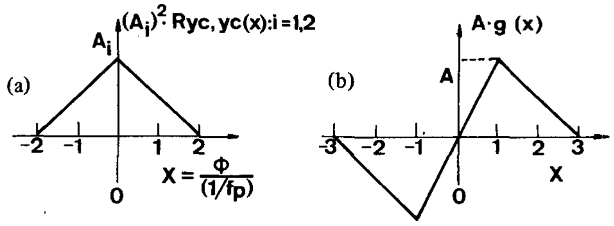


Fig. 4. Correlation function $R_{y'_c,y_c}(x)$ and nonlinearity $g(x)$.

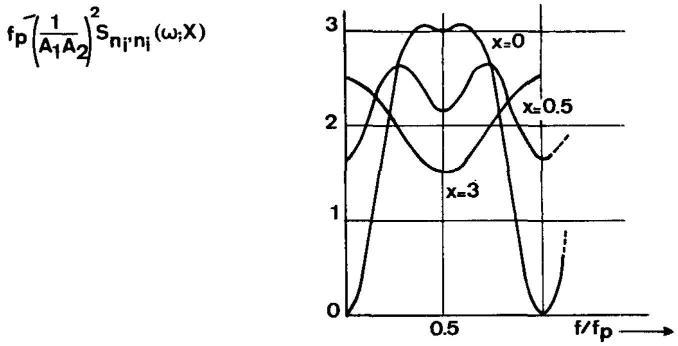


Fig. 5. Normalized power spectrum $S_{n_i,n_i}(\omega;x)$ of the time-continuous process $n_i(t;\phi)$ at the output of the D/A converter.

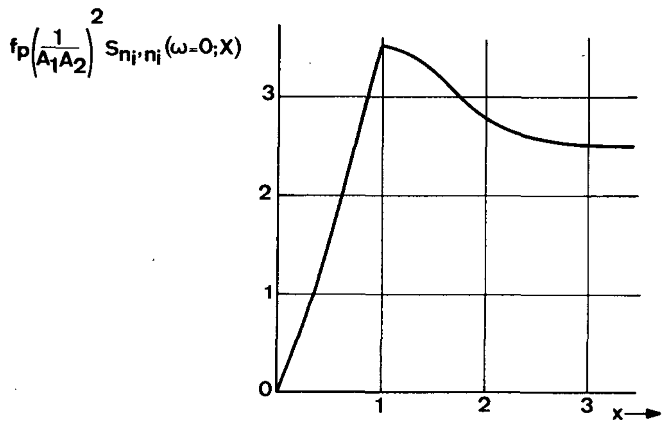


Fig. 6. Normalized intensity coefficient K_{n_i} of the time continuous process $n_i(t;\phi)$.

IV. THE LOOP EQUATIONS

The control law of the variable delay is defined by (5) where (Fig. 3) $u_s(t)$ is given by

$$u_s(t) = F(p)z(t;\phi). \tag{15}$$ [3]

The process $z(t;\ \phi)$ is the time continuous version of $z(k;\ \phi)$ appearing at the output of the D/A converter. If we write $z(t;\ \phi)$ as the sum of expected value and zero mean noise process and insert (15) into (5) then after replacing $\hat{T}$ by $(T - \phi)$ (5) assumes on the form

$$\frac{d\phi}{dt} = -KF(p)[Ag(\phi) + n(t;\phi)] + \frac{dT}{dt}. \tag{16}$$

Expanding the system function $F(p)$ in partial fractions (16) can be written as a system of N first-order differential equations [8].

The mathematical equivalent model of the loop is depicted in Fig. 7. The model has much similarity with the well known baseband model of the PLL with two important differences: 1) the nonlinearity $g(\phi)$ is aperiodic and 2) $n(t;\ \phi)$ depends on ϕ.

V. THE F-P EQUATION

For many applications such as noncontact speed measurement the first-order tracker is the one most often used. For reasons discussed in [2], higher order DLL's are realized by cascading first-order systems. This is in contrast to trackers

[3] The form (15) implies

$$u_s(t) = \int_0^t f(t-x)z(x;\phi)\,dx$$

where $f(t)$ is the impulse response of the loop filter.

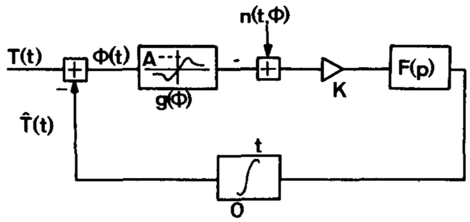


Fig. 7. Mathematically equivalent model of the DLL.

with periodic nonlinearity (PLL, etc.) where the second-order loop is by far the most important one. In the sequel we therefore restrict our discussion to first-order systems. The extension to an Nth-order tracker is straightforward.

The statistics of the process $\phi(t)$ can be determined from a F–P equation provided that we can justify the condition that ϕ is approximately Markov. Stratonovitch shows in [6, p. 89 ff] that an approximate F–P equation can be obtained if the correlation time of the noise process is much shorter than that of $\phi(t)$. No assumption is made on the amplitude distribution, i.e., $n(t;\ \phi)$ need not be Gaussian. This is important in this paper since $n(t;\phi)$ is clearly non-Gaussian. The F–P equation corresponding to (16) with $F(p) = 1, dT/dt = a$ is found to be

$$\frac{\partial}{\partial t}p(\phi;t) + \frac{\partial}{\partial \phi}\left\{K_0(\phi)p(\phi;t) - \frac{1}{2}\frac{\partial}{\partial\phi}[K_{0,0}(\phi)p(\phi;t)]\right\} = 0 \tag{17}$$

with

$$K_0(\phi) = -KAg(\phi) + a + \frac{1}{4}K^2\frac{d}{d\phi}\left(\frac{N(\phi)}{2}\right)$$

$$K_{0,0}(\phi) = K^2\frac{N(\phi)}{2}$$

$$\frac{N(\phi)}{2} = S_{n,n}(\omega = 0; \phi). \tag{18}$$

The intensity coefficient $K_0(\phi)$ is *not* just the restoring force but the sum of restoring force and a term which is formally identical to the Itô correction. Note that this additional term is lost if one formally replaces $n(t;\phi)$ by a delta correlated process.

III. MODELING THE SYSTEM'S BEHAVIOR AS A RENEWAL MARKOV PROCESS

For every physically realizable tracker the pdf $p(\phi;\ t)$ vanishes for $t \to \infty$ if ϕ is allowed to assume any value on the real line, hence ϕ has unbounded variance [2]. This behavior is directly traceable to the "falling out of lock" phenomena associated with tracking systems. Since the nonlinearity $g(\phi)$ tends too rapidly towards zero as $|\phi| \to \infty$, the probability for a trajectory to reach any point on the real line equals one.

Since the steady-state of a system defined in an appropriate manner is of most practical interest, we must determine the pdf of an equivalent process that models the actual behavior of such systems; thus leading to a meaningful result for $t \to \infty$. This can be done as follows: At the beginning of each measurement the DLL has to be brought within the "in-lock" region. This is usually done by sweeping the electronic delay line over the possible range of T. If $d\hat{T}/dt$ is suitably chosen, the DLL will lock when $\hat{T} = T$. The "in-lock" condition is then supervised by a lock detector (Fig. 8) which basically estimates the even correlation function $R_{x,x}(\phi)$ which is maximum for $\phi = 0$. As soon as $R_{x,x}(\phi)$ falls below a certain threshold the system signals "out of lock." Immediately thereafter a new search procedure is started to bring the system back to its "in-lock" region. It has to be kept in mind that the lock detector indicates "out-of-lock" at the points ϕ_L corresponding to $R_{x,x}(\pm\phi_L) = k_1$ only on the average and that there exists for every ϕ a probability that the circuit indicates "out of lock." In the following we neglect this effect and assume a perfect lock-detector signaling "out of lock" exactly at $\phi = \pm\phi_L$. Thus the behavior of the sytem can be modeled in the following way: At time $t = 0$ we start a random process at the stable lock point ϕ_0. At time $t = \tau_{L,1}$ the trajectory reaches either of the two barriers located at $\phi = \pm\phi_L$. Since the mean lifetime of a trajectory for a reasonable loop is much greater than the ... ory is started ... redecessor. A ... wn in Fig. 9. ... s $\tau_{L,K}$ of the individual processes are statistically independent random variables. Therefore, the starting points $t = \tau_{L,1} + \cdots \tau_{L,K}$ form a renewal process. The regenerative process of Fig. 9 obeys in the stationary state a F–P equation of the form [9]

Fundamental falsche Interpretation
der Arbeiten von R.L. Stratonovich.
Kapitaler Unfug !

$$Lp(\phi) = \frac{q(\phi; t = 0)}{E(\tau_L)} \tag{19}$$

with

$$L = \frac{\partial}{\partial\phi}\left[K_0(\phi) - \frac{1}{2}\frac{\partial}{\partial\phi}K_{0,0}(\phi)\right]$$

$p(\phi)$: stationary pdf

$q(\phi; t = 0) = \delta(\phi - \phi_0)$: initial pdf of the single process.

Since all trajectories start at $\phi = \phi_0$, the initial distribution is the Dirac function $\delta(\phi - \phi_0)$. The boundary conditions for ϕ are

$$q(\phi = -\phi_L;\ t) = q(\phi = \phi_L;\ t) = 0. \tag{20}$$

They are of the absorbing type. This means that the sample trajectories are stopped (absorbed) when they first reach one of the barriers located at $\phi = \pm\phi_L$.

IV. ANALYSIS

A. The Stationary pdf of a First-Order Tracker

The stationary pdf is obtained by solving (19) subject to the boundary conditions (20) where the intensity coefficients $K_0(\phi)$ and $K_{0,0}(\phi)$ are given by (18).

For practical purposes it is advantageous to replace ϕ by the dimensionless quantity

$$x = \frac{\phi}{\phi_N} \tag{21}$$

ϕ_N: arbitrary normalization constant.

Without belaboring the details we obtain [2]

$$p(x) = \frac{\alpha_T}{E(\tau_L)} \frac{1}{\sqrt{1/\rho_s \bar{N}(x) + 1/\rho_n}} \exp\left[-\phi_N U_0(x)\right] \cdot \int_{-x_L}^{x} \left[D_0 - u(x' - x_0)\right] \frac{1}{\sqrt{1/\rho_s \bar{N}(x) + 1/\rho_n}} \cdot \exp\left[\phi_N U_0(x')\right] dx' \qquad (22)$$

with

$$1/\rho_s = \frac{2B_L (A_1 A_2)^2}{f_p A^2}; \qquad \bar{N}(x) = f_p \frac{S_{n_i,n_i}(\omega = 0,x)}{(A_1A_2)^2}$$

$$1/\rho_n = \frac{2B_L S_{n_n,n_n}(\omega = 0)}{A^2}; \qquad x_0 = \frac{\phi_0}{\phi_N}. \qquad (23)$$

The parameter α_T is the time constant of the linearized loop ($Ag(\phi) = 1/\phi_N \cdot \phi$) while B_L is the equivalent loop bandwidth

$$1/\alpha_T = 4B_L = \frac{AK}{\phi_N}. \qquad (24)$$

ρ_N is the signal-to-noise ratio of the linearized loop if the intrinsic noise $n_i(t; \phi)$ can be neglected while $\rho_S \cdot 1/\bar{N}(x)$ is the corresponding parameter for the intrinsic noise. By inspection of the definition of $S_{n_i,n_i}(\omega = 0; x)$, one sees that $1/\rho_S \bar{N}(x)$ is independent of the signal power as it should be.

$U_0(x)$ is a potential function defined by the integral

$$U_0(x) = -\frac{1}{\phi_N} \int^{x} \frac{1}{1/\rho_s \bar{N}(x) + 1/\rho_n} \cdot \left[-g(x') + \frac{a\alpha_T}{\phi_N}\right] dx' \qquad (25)$$

while D_0 is a constant of integration determined by the boundary conditions (20)

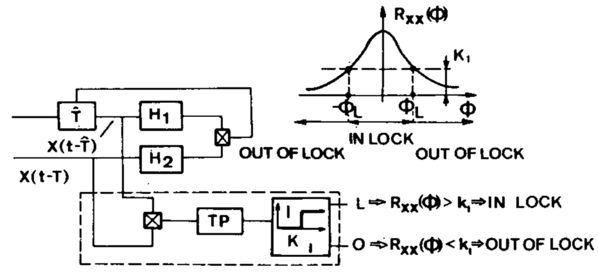
$$D_0 = \frac{\displaystyle\int_{x_0}^{x_{\max}} \exp\left[\phi_N U_0(x)\right] dx}{\displaystyle\int_{x_{\min}}^{x_{\max}} \exp\left[\phi_N U_0(x) dx\right.}. \qquad (26)$$

Example: pdf $p(x)$ and $E(\tau_L)$ of a digital DLL with triangular nonlinearity.

This specific correlation function (see Fig. 4) appears in the DLL used to measure velocity [2]. The D/A converter is assumed to be a first order hold circuit as described in (A9). The normalization constant ϕ_N is given by $\phi_N = 1/f_p$. The pdf $p(x)$ is shown in Fig. 10.

Due to the fact that $S_{n_i,n_i}(\omega = 0,x)$ vanishes for $x = 0$ we observe a sharp peak in $p(x)$ for $x_0 = 0$. For the same reason the variance σ_x^2 is strongly dependent on x_0 if the intrinsic noise $n_i(k; x)$ is dominant.

In order to verify the results of the theory and the validity of the made assumptions, a computer simulation program of the actual loop configuration (not the equivalent model) was written. The results for two runs with $x_0 = 0$ and $x_0 = 0{,}5$ are shown in Fig. 11. The simulation confirms the strong dependence of $p(x)$ on $S_{n_i,n_i}(\omega = 0,x)$. Also, the validity of the assumptions leading to the theoretical model are justified. It is worthwhile to note that the dependence on $S_{n_i,n_i}(\omega = 0,x)$ can be explained only by the nonlinear theory. Finally, the normalized mean time between two failures $E(\tau_L)/\alpha_T$ as a function of ρ_S for several stable lock points x_0 is shown in Fig. 12.



TP: Low Pass Filter

Fig. 8. DLL and associated lock detector.

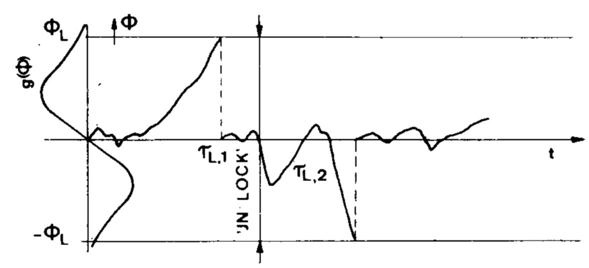


Fig. 9. Sample trajectory of the repeated process when the lock-detector is assumed to work perfectly. The epochs $\tau_{n1}, \tau_{n1} + \tau_{n2}$, etc. indicate when a trajectory first reaches $\pm\ \phi_L$ where the ideal lock-detector signals "out of lock."

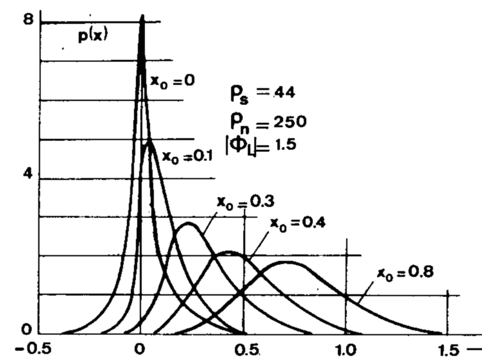


Fig. 10. Probability density function $p(x)$ for the hybrid DLL with triangular nonlinearity.

CONCLUSIONS

A device to track the delay between two versions of a stochastic signal was analyzed. The practical application which motivated this study was speed and distance measurement for nonconventional train systems.

Since phenomena such as falling out of lock are of utmost practical importance, an exact nonlinear analysis of the statistical behavior of the tracking device was necessary. Also, the

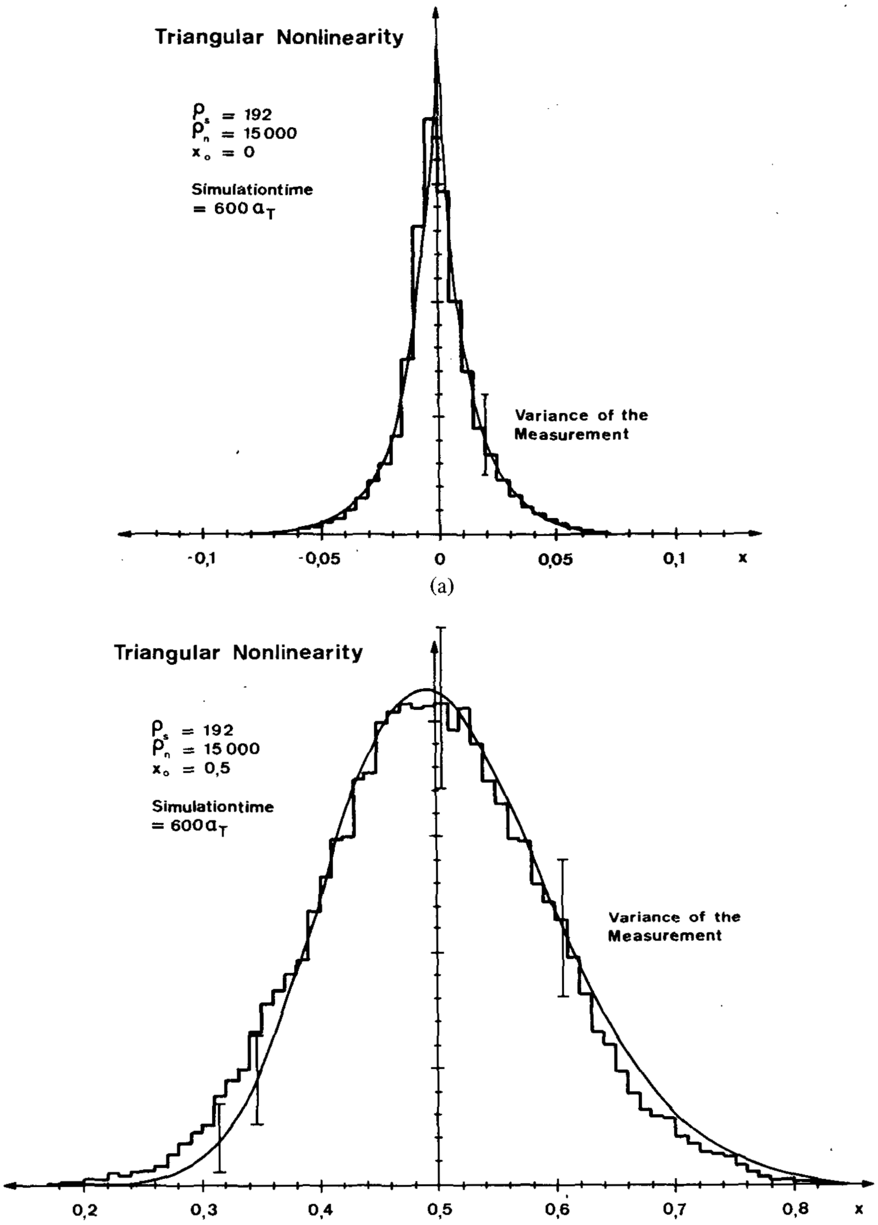


Fig. 11. Experimental and theoretical pdf for hybrid DLL with triangular nonlinearity.

influence of the intrinsic noise which is dependent on ϕ and which is dominant in certain cases can only be understood in the context of a nonlinear theory.

The computer simulation confirms the validity of the assumptions which lead to the model described. It was shown, e.g., that the digital instrumentation can be modeled as an analog device thus permitting the application of the powerful tool of F–P analysis.

APPENDIX A

The most convenient way to compute $S_{n,n}(\omega;\phi)$ is via the correlation function $R_{n,n}(m;\phi)$

$$R_{n,n}(m;\phi) = E[z(k+m;\phi)z(k)]$$
$$= R_{z,z}(m;\phi) - E^2[z(k,\phi)]. \quad \text{(A1)}$$

Note that ϕ plays the role of a parameter. In (A1), by definition, $R_{n,n}(m;\phi)$ is given by

Das ist gar kein Wunder! Simulation und analytische Berechnung basieren nämlich auf ein und derselben falschen Modellierung des physikalischen Prozesses.

(A2)

Of the 16 terms within the braces only 4 are nonzero under the assumptions made. After some algebraic manipulations one finds that $R_{n,n}(m;\phi)$ is the sum of two correlations functions

$$R_{n,n}(m;\phi) = R_{n_n,n_n}(m) + R_{n_i,n_i}(m;\phi). \quad \text{(A3)}$$

The first term $R_{n_n,n_n}(m)$ describes the contribution of the additive noise processes $n_1(k)$,$n_2(k)$. This term vanishes if $n_1(k) = n_2(k) = 0$. The second term is due to the so-called intrinsic or self noise and is defined by

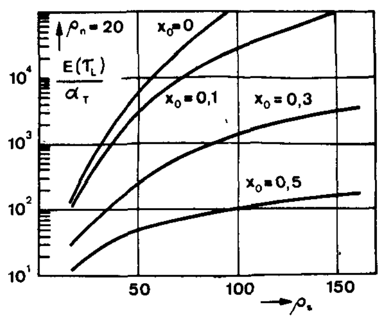


Fig. 12. $E(\tau_L)$ in function of ρ_s for several x_0.

$$R_{n_i,n_i}(m;\phi)=(A_1A_2)^2\cdot E[y_1(k)\cdot y_2(k)y_1(k+m)y_2(k+m)]-E^2[z(k;\phi)]. \quad \text{(A4)}$$

Note that $R_{n_i,n_i}(m;\phi)$ is solely dependent on the signals $y_1(k)$ and $y_2(k)$. In application such as speed measurement [2] this intrinsic noise is dominant and cannot be neglected. Furthermore, $n_i(t;\phi)$ determines the range of useful application of such systems.

The computation of $R_{n_i,n_i}(m;\phi)$ requires knowledge of a four-dimensional pdf which, in general, is unknown. In many cases $x(k)$ and consequently $y_1(k)$, $y_2(k)$ are approximately Gaussian and we find for the configuration of Fig. 3

$$\begin{aligned}R_{n_i,n_i}(m;\phi)=(A_1A_2)^2\cdot&[R_{y_c,y_c}(m+1-\phi)\\&-R_{y_c,y_c}(m-1-\phi)]\cdot[R_{y_c,y_c}(m-1+\phi)\\&-R_{y_c,y_c}(m+1+\phi)]+(A_1A_2)^2\cdot\\&\cdot[-R_{y_c,y_c}(m-2)+2R_{y_c,y_c}(m)\\&-R_{y_c,y_c}(m+2)]\cdot R_{y_c,y_c}(m).\end{aligned} \quad \text{(A5)}$$

It is important to realize that $R_{n_i,n_i}(m;\phi)$ depends on ϕ while $R_{n_n,n_n}(m)$ does not.

Power spectrum and correlation function of the sampled signal are coupled by the relation

$$S_{n,n}(z;\phi)=f_p\cdot\sum_{m=-\infty}^{\infty}R_{n,n}(m;\phi)\cdot z^m$$
$$z=\exp(j\omega\cdot 1/f_p). \quad \text{(A6)}$$

Usually the factor fp is omitted. However, this leads to erroneous results if we pass our sampled and processed signal through a D/A converter and are interested in the power spectrum at the output of the D/A converter.

Of particular interest is the value of the power spectrum in the vicinity of zero since this part of the spectrum cannot be eliminated by the loop filter $F(p)$. Inserting (A5) into (A6) yields for the power spectrum of the intrinsic noise

$$S_{n_i,n_i}(z=1;\phi)=0$$
$$z=1\Leftrightarrow\omega=0;\pm k2\pi f_p. \quad \text{(A7)}$$

Relation (A7) is true if and only if the product $H_1(z)\cdot H_2(1/z)$ is decomposed as in Fig. 3.

Since $\phi=0$ is the stable lock point of the loop (assuming $dT/dt \ll 1$) it is a very desirable property that the power spectrum of the intrinsic noise vanishes at $\omega=0$. Finally, the power spectrum of the output of a D/A converter is given by

$$S_{n,n}(\omega;\phi)=|H_{D/A}(\omega)|^2S_{n,n}[z=\exp(i\omega\cdot 1/f_p);\phi]. \quad \text{(A8)}$$

Example

The system function of a first-order hold circuit is

$$|H_{D/A}(\omega)|^2=\left(\frac{1}{f_p}\right)^2\left(\frac{\sin(\omega 1/2f_p)}{\omega 1/2f_p}\right)^2. \quad \text{(A9)}$$

For $\omega=0$ we find

$$|H_{D/A}(\omega=0)|^2=\left(\frac{1}{f_p}\right)^2. \quad \text{(A10)}$$

Using this result (A10) we find for the power spectrum of the analog signal at the output of the D/A converter

$$S_{n,n}(\omega=0;\phi)=\frac{1}{f_p}\sum_{m=-\infty}^{\infty}R_{n,n}(m;\phi). \quad \text{(A11)}$$

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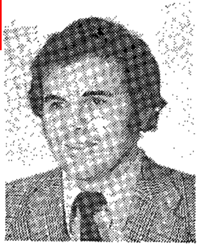

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